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INVESTIGATION OF A TURBULENT BOUNDARY LAYER ON A HYPERSONIC AIRCRAFT MODEL

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An algorithm for calculation of a spatial compressible turbulent boundary layer on the surface of a pointed body is developed. The algorithm is based on the numerical solution of three-dimensional equations and algebraic models of turbulence. The flow around a hypersonic aircraft model is calculated, and the resultant Stanton numbers are compared with experimental data. The influence of the Mach number, the angle of attack, and the Reynolds number on the boundary-layer parameters is studied. It is shown that the change in the location of the transition zone has a weak effect on the skin-friction coefficient in the region of developed turbulent flow.

1. In the development of aviation and spacecraft technology, an important problem is the study of aerodynamic heating of the aircraft surface and the friction forces whose contribution to the total drag is quite significant, especially for a pointed body at small angles of attack. Two approaches are traditionally used to solve this problem: theoretical and experimental. Measurement results are usually considered to be more correct and are used, in particular, for verification of theoretical (numerical) methods, though direct experiments are rare, and the researcher has to choose a certain model of the phenomenon to calculate the desired parameter. The theoretical approach is constructed on the basis of a certain flow model, which fails to take into account all specific features of the phenomenon. In addition, since the use of numerical methods for solving complex systems of equations can introduce its own errors, verification of an algorithm requires a comparison of the results obtained by this algorithm with calculations based on different models and with experiments.

One of the advantages of the theoretical approach is the possibility of calculating all the flow parameters simultaneously, whereas only one parameter is generally measured experimentally. Moreover, in hypersonic flow studies one should take into account that none of the wind tunnels existing in the world reproduces the full-scale stagnation temperature and, hence, does not fully simulate the thermal processes. The advantages of the numerical approach are also the simplicity of variation of the governing parameters (within the validity of the chosen model), the rapid obtaining of new results, and the low cost of calculations as compared with experiments.

The most complete model that describes various flows is the model based on the full Navier–Stokes equations, which were used to obtain the flow around a number of bodies [1–8]. The first two papers are focused on two-dimensional flows around blunted bodies and airfoils, and the remaining papers deal with three-dimensional flows. Isolines and pressure distributions over the body surface are presented in all papers as the numerical results, and only Radespiel and Swanson [2], Herrman et al. [3], and Schröder and Hartmann [5] give skin-friction coefficients or Stanton numbers. The latter, however, are not compared with the results of other authors and experiments, and this is not incidental.

The system of the full Navier–Stokes equations is elliptical, and marching methods of calculation relative to any physical variable are incorrect for it. A fourth variable, the time, is usually added for

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convenience. The problem becomes parabolic and is solved by the pseudo-transient method, but only advanced supercomputers can cope with this four-dimensional problem for spatial bodies.

In solving the Navier–Stokes equations for flows with real Reynolds numbers (10^7–10^8), large gradients of the flow parameters near the wall should be taken into account. A precise calculation of turbulent flow requires not only several dozens of points of the difference scheme across the boundary layer, but also several points (4–6) in the laminar sublayer. Only in this case is it possible to obtain skin-friction coefficients and Stanton numbers on the surface with an acceptable accuracy. However, this condition imposes stringent requirements on the number of steps and extension of the normal coordinate and is not always observed. Obviously, that is why most papers demonstrate the general pattern of the flow and the pressure distribution over the surface, since these results are the least sensitive ones to the mentioned requirement. Practically no one takes the risk of comparing the skin-friction coefficients and Stanton numbers with experimental data in the above-mentioned papers.

Apart from the full Navier–Stokes equations, parabolized (simplified) Navier–Stokes equations where viscous terms along the marching coordinate are omitted [9–12] and viscous shock-layer equations with viscous terms only normal to the surface are left [13–17] are frequently used to calculate spatial flows. The advantage of these approximations (conservation of all terms of the Euler equations) turns out to be simultaneously their drawback. These equations remain elliptical in the near-wall subsonic regions, which allows an upstream propagation of the perturbations. Thus, the marching method can be used only after applying some "regularization" in the near-wall region. It remains unclear, however, what errors are introduced by "regularization" to the flow parameters in this region, in particular, to skin-friction coefficients and Stanton numbers calculated by differentiation of velocity and temperature profiles in the near-wall region.

The requirement concerning the number of points across the turbulent boundary layer and in the laminar sublayer for these models is also imposed in these papers, but not everywhere fulfilled. It seems that the calculated skin-friction coefficients or Stanton numbers are compared with experiments only in [13, 14, 17].

Another drawback of simplified Navier–Stokes equations is the absence of rigorous mathematical justification in the sense of a certain asymptotical theory. In addition, there is some arbitrariness in choosing the marching coordinate and, hence, the probability of some uncertainty in the solution, which cannot be estimated *a priori*. In this aspect, the boundary-layer theory proposed by Prandtl [18] and justified by Van Dyke [19] is a rigorous asymptotic theory for high Reynolds numbers, and the higher the Reynolds number, the more accurate the description of a real flow. Because of their parabolicity, the boundary-layer equations can be solved by the marching method, and the requirement on the number of points in the crossflow direction is quite feasible even for medium-class computers, since the solution is constructed separately in a narrow near-wall region.

The drawback of the boundary-layer model is that it cannot be used independently. The boundary-layer equations require all the flow parameters at the external boundary to be known, except for the normal component of the velocity, which can be found experimentally or numerically from inviscid calculations. The conditions at the external boundary should be determined very accurately, since their error increases with solution of the boundary-layer equations using these conditions. The wetted surface should be also described rather accurately by a certain smooth function with continuous second derivatives, which enter the coefficients of these equations.

Based on the aforesaid, we chose the classical Prandtl model [18] to study the flow around a hypersonic aircraft. In this model, the flow between the body and the shock wave is divided into an inviscid region and a thin boundary layer. The Euler equations were solved for the first region and the spatial boundary-layer equations for the second region.

2. There are some papers devoted to the study of a laminar boundary layer on pointed elliptical cones at high [20–24] and low angles of attack [25, 26]. For these bodies, experimental values of the Stanton number St were obtained [27]. For high angles of attack, the divergence line is typically located on the cone surface in the plane of symmetry, where the maximum values of the skin-friction coefficient c_f and St are observed. For small angles of attack, the divergence line can be located outside the plane of symmetry, and the maximum values of c_f and St coincide with neither of them. A spatial flow around a pointed bielliptical body was

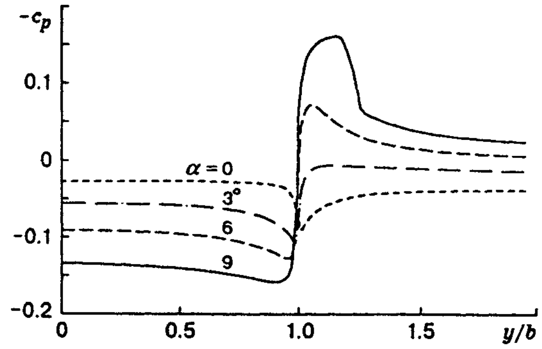

Fig. 1

calculated by Vetlutsky [28], where it was shown that the divergence surface can disappear in the boundary layer, which is caused by the shift of the pressure maximum from the plane of symmetry to the side.

Some papers are devoted to a laminar boundary layer on flat and shaped delta wings [29–36]. A turbulent boundary layer of the same geometry was calculated by Vetlutsky and Poplavskaya [37, 38]. The effect of the governing parameters was studied there, and the contribution of the friction forces to the total drag of the wing was evaluated. A turbulent boundary layer was experimentally studied for delta wings with sharp [39] and blunted [40] leading edges.

A calculation algorithm for laminar and turbulent boundary layers on a pointed fuselage-shaped body was proposed in [41–43]. Experimental studies of a turbulent boundary layer on ogive-cylinder bodies at incidence were described in [44–47]. For the first time, a supersonic flow around the ELAC 1 model was calculated using the viscous shock-layer model [15], but the distributions of c_f and St were not presented. The Stanton numbers on the surface of this model were measured in a shock tube for the Mach number M = 7 [48] and in a supersonic wind tunnel for Mach numbers M = 1.5–2.5 [49].

3. The present study of a turbulent boundary layer was conducted on the forebody of the hypersonic aircraft model ELAC 1, which is a sharp bielliptical cone with a semi-apex angle of 15° in the horizontal plane. The ratio of the cross-sectional ellipsis was 1 : 4 for the leeward side and 1 : 6 for the windward side, which corresponds to the semi-apex angles of 3.83 and 2.56° in the vertical plane, respectively.

The inviscid flow behind the shock wave was calculated using the Euler equations written in the divergent form and presented in the difference form by the method of finite volumes proposed by Roe [50]. In spatial approximation of the derivatives, we used the flux difference splitting approach [51] with account of the properties of disturbance propagation. The fluxes at the cell faces are calculated using van Leer's monotone upstream-centered schemes for conservation laws, which ensured the second-order accuracy and the suppression of oscillations at discontinuities (shock waves). In our calculations, we used a difference scheme fitted with the body surface, which was proposed by Hänel et al. [15] for solving the viscous shock-layer equations. The system of difference equations was solved by a nonlinear multigrid method with the Gauss–Seidel relaxation for smoothing. The calculations were performed on a Convex C3820 mini-supercomputer for Mach numbers $M = 2.5$, 4, and 7 and angles of attack from 0 to 15°.

Figure 1 shows the distribution of the pressure coefficient c_p on the body surface in the cross section $\xi/L = 0.36$ for $M_\infty = 2.49$ and different angles of attack. Here and in what follows, the abscissa axis shows the ratio of the transverse Cartesian coordinate y to the local half-span b with continuation behind the leading edge to the leeward side, where the value of y/b varies from 1 to 2. The pressure is almost constant on the windward and leeward sides, it changes abruptly only near the leading edge. For $\alpha = 9°$, a barrel shock wave is formed on the leeward side ($y/b \cong 1.2$).

The boundary-layer calculations are performed using the equations of a three-dimensional compressible boundary layer, which are written in the nonorthogonal coordinate system (ξ, η, ζ) fitted with the body surface. The coordinate ξ is measured from the body tip along some axis, η is the distance along the normal to the surface, and ζ is the meridional angle in the cross section [41–43]. It is assumed that the body surface near the tip can be approximated by a conical surface. In this case, the boundary-layer equations allow a self-similar solution, which depends on the variables ζ and $\eta/\sqrt{\xi}$ [25, 26]. This solution can be used as the initial data for the problem as a whole. The flow parameters obtained in the inviscid calculation of the flow around this body were set at the external boundary. The symmetry conditions were applied in the plane $\zeta = 0$ if it was a divergence surface; otherwise, no conditions were needed. The other boundary $\zeta = \zeta_k$ was always chosen so that the gas left the computational domain through this boundary; therefore, no conditions were set there. The boundary-layer equations were solved numerically using an absolutely stable second-order difference scheme, which was not fitted with the divergence surface [52].

For calculation of the laminar, transitional, and turbulent flows, the equations contained the total viscosity and thermal conductivity, in which turbulent coefficients were added to laminar coefficients with a certain intermittency factor Γ. Four different algebraic models of turbulence were used for turbulent viscosity. The skin-friction coefficients calculated by each model were compared previously; the results were very close to each other [43]; hence, all these models can be used. It was also noted there that the minimum number of nodes of the difference grid normal to the surface in a turbulent flow should be equal to 60. In the present work, it was equal to 60 and 80.

The value of Γ varied from 0 to 1 depending on the parameter $A = \mathrm{Re}_{\delta^{**}}/\exp(0.2\mathrm{M}_e)$, where $\mathrm{Re}_{\delta^{**}}$ is the Reynolds number based on the momentum thickness and M_e is the local Mach number at the outer edge of the boundary layer [38]. In the cited paper, the parameters of the transition beginning A_1 and end A_2 were chosen from the experiments for a delta wing and were equal to 150 and 350, respectively. Exactly these values of the transition parameters were used in the present calculations. The calculation algorithm for a turbulent boundary layer was tested on ogive-cylinder bodies at incidence [42, 43], where it was compared with the results of four experimental papers [44–47]. The calculated and experimental data are in good agreement for the velocity profiles and local Mach numbers, and also the distributions of the skin-friction coefficients and Stanton numbers.

The study of the boundary layer on the ELAC 1 model began from a comparison with experiments performed for this model. The Stanton number distributions

$$\mathrm{St} = q\Big|_{\eta=0}\Big/\rho_\infty U_\infty c_p(T_0 - T_w)$$

measured in a shock tube for $\mathrm{M}_\infty = 7$, angles of attack from 0 to 15°, and the Reynolds number based on the model length $\mathrm{Re}_X = 1.1 \cdot 10^6$ are given in [48]. The calculation results obtained in the cross sections $x/X = 0.3$, 0.45, and 0.6 using the algorithm described are also presented there. It is shown that the difference is less than 16% and the boundary layer is laminar on the entire surface.

The other series of experiments on the ELAC 1 model was conducted in a supersonic wind tunnel of the Aachen Institute of Aerodynamics (Germany) for $\mathrm{M}_\infty = 1.5$–2.5 and $\alpha = 0$–9° [49]. The specific feature of this wind tunnel is that it has no high-pressure plenum chamber, the air is sucked from the atmosphere, and the gas is exhausted to a vacuum chamber; thus, it should be expected that the level of perturbations in the test section of this wind tunnel is lower than in ordinary wind tunnels. For comparison with these experiments, the parameters of the laminar–turbulent transition A_1 and A_2 were chosen to be 250 and 550, respectively. In addition, the stagnation temperature of the flow was close to the temperature of the model surface. Hence, the denominator in the expression for the Stanton number was close to zero, and the calculated parameter was

$$\mathrm{St}^+ = q\Big|_{\eta=0}\Big/\rho_\infty U_\infty c_p(T_w - T_\infty).$$

The calculated values of St^+ (solid curves) and the experimental results (points) for $\mathrm{M}_\infty = 2.5$, $\mathrm{Re}_X = 7.1 \cdot 10^6$, and different angles of attack in the cross section $x/X = 0.3$ are compared in Fig. 2. The turbulent region is located on the windward side up to the values $y/b = 0.5$–0.7, and then the flow is laminar up to the leading edge. As the angle of attack increases, the increase in St^+ in the turbulent region as compared with the laminar region becomes more and more noticeable. A comparison shows satisfactory agreement between the theoretical and experimental results: the difference is less than 16%.

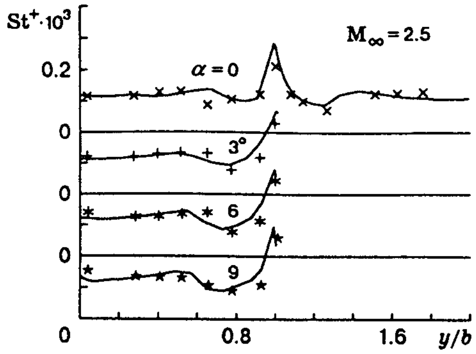

Fig. 2

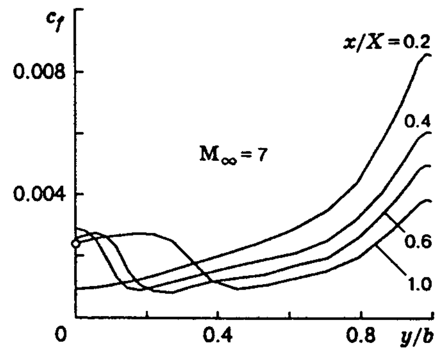

Fig. 3

4. The boundary-layer study was performed for an enthalpy factor $H_w/H_\infty = 0.565$ and $\mathrm{Re}_X = 3\cdot10^6$. The absolute value of the skin-friction coefficient c_f versus y/b in several cross sections is shown in Fig. 3 for $\alpha = 5°$ and $M_\infty = 7$. It can be seen that the flow in the initial cross section $x/X = 0.2$ is laminar everywhere; then it becomes turbulent near the plane of symmetry and the turbulent region extends to the leading edges and reaches the value $y/b = 0.3$ in the last cross section. A narrow region of the laminar–turbulent transition is adjacent to the turbulent region, and then the flow is laminar everywhere up to the leading edges.

The decrease in the Mach number to 4 induced a noticeable region of turbulent flow near the plane of symmetry already in the cross section $x/X = 0.2$. Moving along the model axis, the turbulent region becomes wider, and in the last cross section it approaches the value $y/b = 0.8$. The flow remains laminar near the leading edges on the whole model.

For $M_\infty = 2.5$, the turbulent region occupies almost half of the ELAC 1 surface already in the cross section $x/X = 0.2$; then it extends practically over the entire surface, and in the last cross section the boundary layer remains laminar only on the leading edge.

The ELAC 1 model is a shaped delta wing with rounded edges, which allows a comparison of the boundary-layer parameters for this model with the calculations for a delta wing with sharp leading edges. Nevertheless, it should be noted that the flow patterns for these two cases are significantly different, especially near the leading edges. For the ELAC 1 model, the shock wave is always detached; for a wing with sharp edges, the shock wave is attached at a certain angle of attack. However, as the stream filaments move away from the edges, their effect becomes weaker, and it can be expected that the boundary-layer parameters will be close to each other in the vicinity of the plane of symmetry.

To compare the above-said, the boundary-layer calculations were performed for a flat delta wing with sharp leading edges for $M_\infty = 7$, $\alpha = 5$, 10, and 15°, and equal values of the remaining governing parameters as for the ELAC 1 model. The calculation algorithm was proposed by Vetlutsky and Poplavskaya [37, 38]. Since the local angle of attack in the plane of symmetry for ELAC 1 differs from the angle of attack for a flat wing by 2.58°, the latter results were interpolated to the same local angle of attack. A comparison of the Stanton numbers and the skin-friction coefficients for three angles of attack and $y/b = 0$ shows that the difference is less than 3%. In Fig. 3, the results of interpolation for a triangular plate are marked by circles. This agreement between the calculation results obtained using two different algorithms and codes is one more confirmation of their reliability.

Kovaleva et al. [39, 40] measured the Stanton numbers in longitudinal sections parallel to the plane of symmetry at a distance of y_1/b_1 from it (b_1 is the half-span of the model). Figure 4 shows the calculated values of c_f versus x'/X for $M_\infty = 4$ and $\alpha = 5°$ in the cross sections $y_1/b_1 = 0.2$, 0.4, and 0.6 (here and in what follows, the x' coordinate is laid off from the leading edge). Since the body considered is close to a flat plate, the pressure on the lower surface varies weakly, except for the vicinity of the leading edges (see Fig. 1). Therefore, the boundary layer in longitudinal sections develops in a similar way. A small difference is observed only in the transitional region, but it practically disappears in the zone of developed turbulent

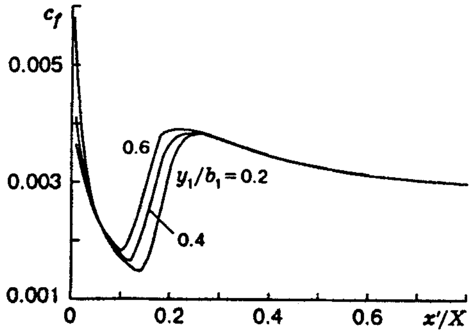

Fig. 4

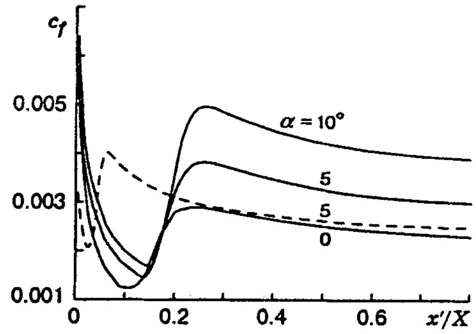

Fig. 5

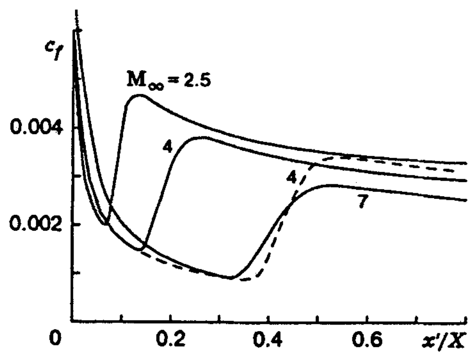

Fig. 6

flow. It follows from here that the laws established for one longitudinal section can be extended to the entire windward surface of the model.

The effect of the angle of attack on the boundary-layer flow is shown in Fig. 5, which shows the dependence of c_f on the coordinate x'/X for $M_\infty = 4$, $Re_X = 3 \cdot 10^6$, and three values of α in the cross section $y_1/b_1 = 0.2$. As the angle of attack increases, the transition region moves away from the leading edge, and the values of c_f in the turbulent region increase significantly. Such a regularity was also observed in experiments [39, 40]. The calculation results for $M_\infty = 4$, $\alpha = 5°$, and $Re_X = 1 \cdot 10^7$ are shown by the dashed curve. As could be expected, the region of the laminar-turbulent transition shifts noticeably toward the leading edge. The value of c_f in the developed turbulent region decreases by approximately 20%, which is not unexpected, since the dependence on the Reynolds number is rather weak for $Re \sim 10^7$.

The skin-friction distributions c_f for $\alpha = 5°$, $Re_X = 3 \cdot 10^6$, and different values of M_∞ in the cross section $y_1/b_1 = 0.2$ are shown by the solid curves in Fig. 6. It can be seen that the coordinate of the beginning of transition increases by an order of magnitude as the Mach number increases by a factor of three. In addition, the twofold difference in c_f in the laminar region decreases approximately to 20% in the region of developed turbulent flow. A similar dependence on the Mach number was noted in measuring the Stanton numbers [39, 40].

In each calculation of a turbulent boundary layer, the question about the location of the transition region and its effect on the calculation results in the turbulent region remains open; therefore, the variant for $M_\infty = 4$ was repeated, but the transition parameters A_1 and A_2 were chosen to be 250 and 550, respectively, as in the comparison with the experiments of the Aachen Institute of Aerodynamics [49]. The resultant values of c_f are shown by the dashed curve in Fig. 6. As could be expected, the beginning of transition shifts significantly more downstream, and the values of c_f in this region are noticeably different. However, in the region of developed turbulent flow this difference is roughly 5%. Thus, the error in specifying the transition region has only a weak effect on the boundary-layer parameters in the developed turbulent region.

5. It is shown in the paper that three-dimensional boundary-layer equations have a number of advantages over the full and parabolized Navier–Stokes equations. Only by using these equations can the friction forces and aerodynamic heating of the surface of spatial bodies be exactly calculated without using supercomputers.

The calculation algorithm for a three-dimensional boundary layer, which includes the suggested criterion of the laminar–turbulent transition, was fundamentally tested at first by comparing the calculated and experimental results. A subsequent study performed for a hypersonic aircraft model using the code proposed, allowed us to obtain data on the effect of the Mach number and the angle of attack on the flow parameters. An extension of the laminar–turbulent transition region from the plane of symmetry to the leading edges in the downstream direction is shown. The lower the Mach number, the faster this extension. An increase in the angle of attack leads to an increase in the skin-friction coefficient and Stanton number, whereas the coordinates of the transition region remain almost unchanged. A decrease in the Mach number also causes an increase in the above-mentioned parameters in the region of developed turbulent flow, but this increase is less significant. However, the transition region shifts substantially upstream in this case.

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